1. (5 points) Consider the function $f(x) = 3x^2 + 9x - 84$. Find the zeros of $f(x)$

$0 = 3(x^2 + 3x - 28)$

$0 = 3(x+7)(x-4)$

$0 = x+7$ | $0 = x-4$

$x = -7$ | $x = 4$

2. (4 points) Consider the function $g(x) = 4x^2 - 24x + 40$. Is $g(x)$ concave up, concave down, neither? State why.

CCU $a = 4 > 0$.

3. (6 points) The point $(4,5)$ is on the graph of $f(x)$. What point must be on the graph of...

(a) $g(x) = f(x-2)+1$

$(6,6)$

(b) $h(x) = f(x+3) - 8$

$(1,-3)$

4. (8 points) The tuition for Wilen University is dependent on the number of credit hours a student is taking in a given semester. If a student is taking less than 12 credit hours, the student must pay \$500 per credit hour along with an additional fee of \$3000. If a student is taking 12 or more credit hours, they must pay 400 per credit hour with along with an additional fee of \$2500. The university will not allow any student to take more than 24 credit hours in a semester.

(a) Find a piece-wise formula for the tuition, T, for a student taking C credit hours.

$$T = \begin{cases} 3000 + 500C & 0 \leq C \leq 12 \\ 2500 + 400C & 12 \leq C \leq 24 \end{cases}$$

(b) What is the domain of this function?

$[0,24]$ or $(0,24]$

or $[1,24]$ or $x = 0, 1, \ldots, 24$

5. (8 points) The profits of the Gansbert Biological Institute (GBI) is directly proportional to the square root of the amount of money that it spends on advertising. The profit of GBI will be \$50,000 if they spend \$10,000 on advertising.

(a) Write a formula for the profit, P, as a function of the amount spent on advertising, A.

$P = K\sqrt{A} \rightarrow 50{,}000 = K\sqrt{10000}$

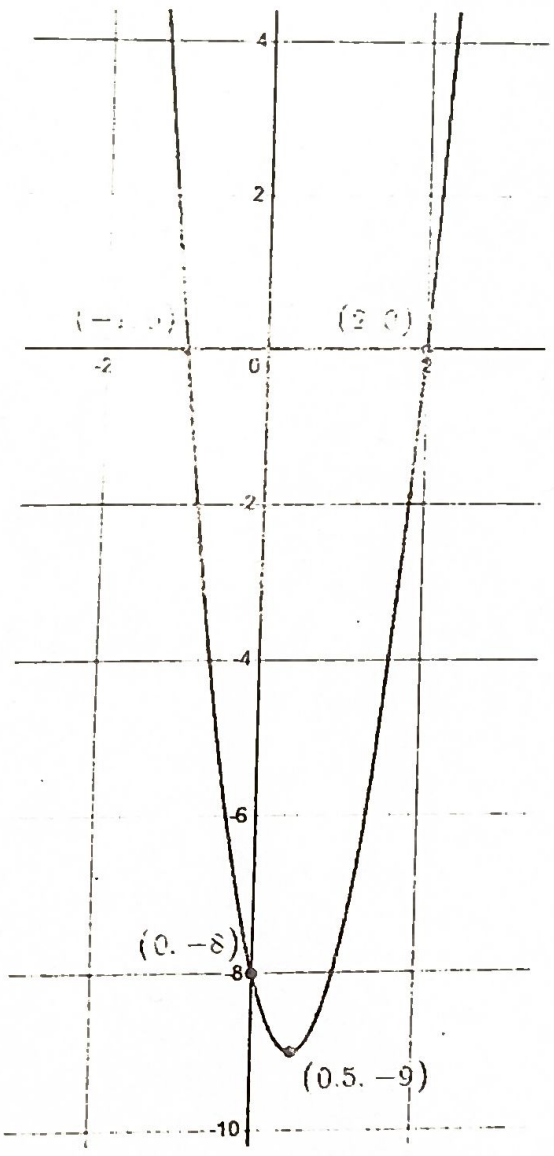
$K = 500$

$P = 500\sqrt{A}$

(b) Find the profit if \$360,000 is spent on advertising

300,000

6. (6 points) Find the formula for the quadratic function $f(x)$ graphed below



$y = a(x+1)(x-2)$

$-8 = a(1)(-2)$

$a = 4$

$y = 4(x+1)(x-2)$

OR

$y = 4(x-0.5)^2 - 9$

7. (9 points) The population of a town is 50,000 in the year 2025. Recall that a linear function has a general form of $P = mt + b$ and an exponential function has a general form of $P = a \cdot b^t$.

(a) If the population of the town is increasing by 300 per year, find a formula for the function $P(t)$, the population t years after 2025.

$P = 50,000 + 300t$

(b) If the population is instead decreasing by 8% per year, find a formula for the function $P(t)$, the population t years after 2025.

$P = 50,000(0.92)^t$

(c) Using your formula from part b, find the year the population will hit 20,000. Round to the nearest whole number.

$t \approx 10.99 \rightarrow 11$

So, 2036

8. (6 points) Otto decides to start to wind down his business selling landline phones. The profits of his business are decreasing at a continuous rate of 15% per year. Find the time it takes for profits to become half of their value today.

$$\frac{\ln(\frac{1}{2})}{-0.15} = 4.621$$

9. (5 points) Determine if the function below is linear, exponential, or neither. Then, find a formula for the function or explain why it cannot be done.

x	1	3	5	7
$f(x)$	20	5	$\frac{5}{4}$	$\frac{5}{16}$

$40 \cdot \left(\frac{1}{2}\right)^x$

10. (10 points) Lilyanna opens a bank account with an initial deposit of \$30,000. It earns interest at a nominal rate of 9% per year.

(a) Find the balance of their account after 8 years if interest is compounded as follows.

(i) Annually (once per year).

\$59,776.88

(ii) Monthly (12 times per year).

\$61,467.64

(iii) Continuously.

\$61,633.00

(b) Of the three cases listed in part (a) of this question, which has the highest effective rate? That is, which account increases the most over the course of one year?

Case (iii)

11. (5 points) Let $f(x) = 3x - 4$, $g(x) = -8x + 1$ and $h(x) = \ln(x)$. Find the following, and simplify your answers completely:

(a) $g(f(5))$

-87

(b) $h(f(x))$

$\ln(3x-4)$

12. (10 points) Consider the exponential function $Q = 55e^{0.72t}$.

(a) Determine if this function displays exponential growth or decay.
Circle one: **exponential growth** or **exponential decay**. Explain your answer in a sentence.

growth, positive growth rate

(b) Give the initial value for this function.

55

(c) Give the continuous growth rate for this function. Write your answer as a percentage.

72%

(d) Give the domain for the given function.

$(-\infty, \infty)$

(e) Give the range for the given function.

$(0, \infty)$

(f) Write the given function in the form $Q = ab^t$.

$Q = 55(2.05)^t$

13. (7 points) Assume $R = f(t) = 1500(1.65)^t$ is the revenue of a business t years after 2025

(a) Evaluate $f(4)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

$11,118.01

The revenue in 2029 is $11,118.01.

(b) Find a formula for $f^{-1}(R)$ in terms of R. Give an exact answer.

$$t = \frac{\ln\left(\frac{R}{1500}\right)}{\ln(1.65)}$$

(c) Evaluate $f^{-1}(10000)$. Round to the nearest whole number

$3.79 \approx 4$

(d) Describe in words what the quantity you found in part c) represents. Write your answer in a complete sentence with units.

The revenue will hit $10,000 in 2029

14. (6 points) Calculate the following limits

(a) $\lim_{x \to -\infty} \frac{4}{x^2}$

0

(b) $\lim_{x \to \infty} (3x - 5x^2)$

$-\infty$